ON SOME PROPERTIES OF THE STATE OF STRESS OF A THIN ELASTIC LAYER

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A construction on the basis of an asymptotic method is given in [1 and 2] for the two-dimensional equations of the theory of thin elastic plates, where the elasticity theory equations in terms of stresses and the Hooke's law relationships are taken as initial [starting] equations. It was assumed in investigating the internal state of stress(*) in [1 and 2] that the displacements and stresses have definite orders, which was confirmed by the case of bending and symmetric deformation of a homogeneous plate. But the state of stress in a relatively weak elastic layer compressed between stiffer layers does not fit in within the framework of this assumption. The need to investigate such states of stress arises in examining weak layers in multilayer plates.

An investigation of the internal state of stress of a thin layer is conducted herein by the method of asymptotic integration of the Lamé equations, whereby no assumptions are made on the order of the surface loading, and it is just considered that the displacements in the plane of the layer is an order less than the displacements out of its plane. This allows a more general asymptotic solution of the equations of elasticity theory to be obtained for the internal state of stress than in [1 and 2], and in a form suitable to describe the behavior of a homogeneous plate subjected to arbitrary surface loading, as well as the behavior of a layer compressed by stiffer layers (weak layers in multilayer plates).

Let us note that a complete investigation of the internal state of stress of a thin plate provides for both the construction of the differential equations, and for the formulation of the appropriate boundary conditions [3]. This latter is associated with the study of the states of stress of boundary layers; they are not considered herein. Hence, the conducted investigations concern only the question of obtaining the differential equations of the internal state of stress.

1. Let us assume that the coordinate plane α, β of an orthogonal curvilinear coordinate system is parallel to the middle plane of the plate. We take the Lamé equations in a form explicitly indicating that its coefficients are independent of the elastic modulus E are expressed only in terms of the Poisson coefficient ν.

$$\begin{aligned}&\frac{1-\nu}{1-2\nu}\frac{1}{H_\alpha}\frac{\partial}{\partial\alpha}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\alpha}(H_\beta u_\alpha)+\frac{\partial}{\partial\beta}(H_\alpha u_\beta)\right]+\frac{\partial u_\gamma}{\partial\gamma}\right\}-\\&\quad-\frac{1}{2H_\beta}\frac{\partial}{\partial\beta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\alpha}(H_\beta u_\beta)-\frac{\partial}{\partial\beta}(H_\alpha u_\alpha)\right]\right\}+\\&\quad+\frac{1}{2}\frac{\partial^2 u_\alpha}{\partial\gamma^2}-\frac{1}{2H_\alpha}\frac{\partial^2 u_\gamma}{\partial\alpha\,\partial\gamma}=0\qquad(\alpha\beta)\end{aligned}\tag{1.1}$$

*) This state of stress is called fundamental in [1 and 2]

$$\frac{1-\nu}{1-2\nu}\frac{\partial}{\partial\gamma}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\alpha}(H_\beta u_\alpha)+\frac{\partial}{\partial\beta}(H_\alpha u_\beta)\right]+\frac{\partial u_\gamma}{\partial\gamma}\right\}-$$

$$-\frac{1}{2H_\alpha H_\beta}\left\{\frac{\partial}{\partial\alpha}\left[H_\beta\frac{\partial u_\alpha}{\partial\gamma}-\frac{H_\beta}{H_\alpha}\frac{\partial u_\gamma}{\partial\alpha}\right]+\frac{\partial}{\partial\beta}\left[H_\alpha\frac{\partial u_\beta}{\partial\gamma}-\frac{H_\alpha}{H_\beta}\frac{\partial u_\gamma}{\partial\beta}\right]\right\}=0$$

Here u_α, u_β, u_γ are components of the displacement vector, and H_α, H_β are Lamé parameters (*). The symbol $(\alpha\,\beta)$ means that there is another relationship obtained by interchanging the α and β.

Let us introduce the nondimensional quantities

$$v_\alpha=\frac{u_\alpha}{h},\quad v_\beta=\frac{u_\beta}{h},\quad v_\gamma=\frac{u_\gamma}{h},\quad \xi=\frac{\alpha}{l},\quad \eta=\frac{\beta}{l},\quad \zeta=\frac{\gamma}{h} \tag{1.2}$$

Here $2h$ is the layer thickness, l the characteristic dimension of the deformation. Let us assume the relative layer thickness $\varepsilon = h/l$ to be small.

We seek the solution of the system of equations obtained from (1.1) by passing to the nondimensional variables (1.2), as

$$v_\alpha=\varepsilon^{\varkappa+1}\sum_{s=0}\varepsilon^s v_\alpha^{(s)}\quad(\alpha\beta),\qquad v_\gamma=\varepsilon^{\varkappa}\sum_{s=0}\varepsilon^s v_\gamma^{(s)} \tag{1.3}$$

If (1.3) is substituted into the Lamé equations transformed to nondimensional form, and terms in identical powers of ε are equated to zero, we then obtain the following Eqs. to determine $v_\alpha^{(s)}$, $v_\beta^{(s)}$, $v_\gamma^{(s)}$:

$$\frac{1}{2(1-2\nu)}\frac{1}{H_\alpha}\frac{\partial^2 v_\gamma^{(s)}}{\partial\xi\,\partial\zeta}-\frac{1}{2}\frac{\partial^2 v_\alpha^{(s)}}{\partial\zeta^2}=$$

$$=-\frac{1-\nu}{1-2\nu}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\alpha^{(s-2)})+\frac{\partial}{\partial\eta}(H_\alpha v_\beta^{(s-2)})\right]\right\}+$$

$$+\frac{1}{2H_\beta}\frac{\partial}{\partial\eta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\beta^{(s-2)})-\frac{\partial}{\partial\eta}(H_\alpha v_\alpha^{(s-2)})\right]\right\}\qquad(\alpha\beta)$$

$$\frac{1-\nu}{1-2\nu}\frac{\partial^2 v_\gamma^{(s)}}{\partial\zeta^2}=-\frac{1}{2(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial^2}{\partial\xi\,\partial\zeta}(H_\beta v_\alpha^{(s-2)})+\frac{\partial^2}{\partial\eta\,\partial\zeta}(H_\alpha v_\beta^{(s-2)})\right]-$$

$$-\frac{1}{2H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}\frac{H_\beta}{H_\alpha}\frac{\partial v_\gamma^{(s-2)}}{\partial\xi}+\frac{\partial}{\partial\eta}\frac{H_\alpha}{H_\beta}\frac{\partial v_\gamma^{(s-2)}}{\partial\eta}\right] \tag{1.4}$$

The right sides of these equations are expressed in terms of the $(s-2)$-th approximation, which should be considered known in determining the s-th approximation. Eqs. (1.4) can be integrated with respect to ζ, resulting in the solution

$$v_\alpha^{(s)}=\sum_{k=0}^{r+1}\zeta^k v_{\alpha k}^{(s)}\quad(\alpha\beta),\qquad v_\gamma^{(s)}=\sum_{k=0}^{r}\zeta^k v_{\gamma k}^{(s)} \tag{1.5}$$

where

$$r=\begin{cases}s\ , & \text{if}\quad s-\text{ is even}\\ s-1, & \text{if}\quad s-\text{is odd}\end{cases} \tag{1.6}$$

The quantities $v_{\alpha k}^{(s)}$, $v_{\beta k}^{(s)}$, and $v_{\gamma k}^{(s)}$ are functions just of ξ, η; the subscript k on these quantities indicates the power of ζ, for which this quantity is a factor in (1.5). The coefficients of ζ^k in (1.5) are connected by differential dependences which can be obtained if (1.5) is substituted into (1.4) and terms with identical powers of ζ are equated to zero. These dependences are

for $k \geqslant 1$

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*) In [2] cited above, the H_α, H_β denote quantities reciprocal to the Lamé parameters.

$$v_{\alpha,\,k+2}^{(s)}=\frac{1}{(k+1)(k+2)}\left\{-\frac{3-2\nu}{2(1-\nu)}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha k}^{(s-2)})+\right.\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial\eta}(H_\alpha v_{\beta k}^{(s-2)})\right]\right\}+\frac{1}{H_\beta}\frac{\partial}{\partial\eta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\beta k}^{(s-2)})-\frac{\partial}{\partial\eta}(H_\alpha v_{\alpha k}^{(s-2)})\right]\right\}+$$

$$\left.+\frac{1}{2(1-\nu)}\frac{1}{k}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\nabla v_{\gamma,\,k-1}^{(s-2)}\right\}\qquad(\alpha\beta)\qquad(1.7)$$

for $k\geqslant 0$

$$v_{\gamma,\,k+2}^{(s)}=\frac{1}{(k+1)(k+2)}\left\{-\frac{1}{2(1-\nu)}(k+1)\frac{1}{H_\alpha H_\beta}\times\right.$$

$$\left.\times\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha,\,k+1}^{(s-2)})+\frac{\partial}{\partial\eta}(H_\alpha v_{\beta,\,k+1}^{(s-2)})\right]-\frac{1-2\nu}{2(1-\nu)}\nabla v_{\gamma k}^{(s-2)}\right\}$$

We moreover obtain

$$v_{\alpha 2}^{(s)}=\frac{1-\nu}{1-2\nu}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha 0}^{(s-2)})+\frac{\partial}{\partial\eta}(H_\alpha v_{\beta 0}^{(s-2)})\right]\right\}+\qquad(1.8)$$

$$+\frac{1}{2H_\beta}\frac{\partial}{\partial\eta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\beta 0}^{(s-2)})-\frac{\partial}{\partial\eta}(H_\alpha v_{\alpha 0}^{(s-2)})\right]\right\}-\frac{1}{2(1-2\nu)}\frac{1}{H_\alpha}\frac{\partial v_{\gamma 1}^{(s)}}{\partial\xi}$$

The relationships (1.7) are of recurrent nature; they permit determination of $v_{\alpha k}^{(s)}$, $v_{\beta k}^{(s)}$ (for $k\geqslant 3$) and $v_{\gamma k}^{(s)}$ (for $k\geqslant 2$) in terms of the $(s-2)$-th approximation.

2. Let us determine the stresses corresponding to the displacements (1.3). If expressions for the strain components are substituted in the Hooke's law relationship, and the transformation is made to nondimensional quantities (1.2), we then obtain

$$\frac{1}{E}\sigma_{\alpha\alpha}=\varepsilon\left\{\frac{\nu}{(1+\nu)(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\alpha)+\frac{\partial}{\partial\eta}(H_\alpha v_\beta)\right]+\right.$$

$$\left.+\frac{1}{1+\nu}\left(\frac{1}{H_\alpha}\frac{\partial v_\alpha}{\partial\xi}+\frac{1}{H_\alpha H_\beta}\frac{\partial H_\alpha}{\partial\eta}v_\beta\right)\right\}+\frac{\nu}{(1+\nu)(1-2\nu)}\frac{\partial v_\gamma}{\partial\zeta}\qquad(\alpha\beta)\qquad(2.1)$$

$$\frac{1}{E}\sigma_{\alpha\beta}=\varepsilon\frac{1}{2(1+\nu)}\left[\frac{H_\beta}{H_\alpha}\frac{\partial}{\partial\xi}\left(\frac{v_\beta}{H_\beta}\right)+\frac{H_\alpha}{H_\beta}\frac{\partial}{\partial\eta}\left(\frac{v_\alpha}{H_\alpha}\right)\right]$$

$$\frac{1}{E}\sigma_{\alpha\gamma}=\varepsilon\frac{1}{2(1+\nu)}\frac{1}{H_\alpha}\frac{\partial v_\gamma}{\partial\xi}+\frac{1}{2(1+\nu)}\frac{\partial v_\alpha}{\partial\zeta}\qquad(\alpha\beta)$$

$$\frac{1}{E}\sigma_{\gamma\gamma}=\varepsilon\frac{\nu}{(1+\nu)(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\alpha)+\frac{\partial}{\partial\eta}(H_\alpha v_\beta)\right]+\frac{1-\nu}{(1+\nu)(1-2\nu)}\frac{\partial v_\gamma}{\partial\zeta}$$

Substituting the expansions (1.3) into these relationships, and collecting terms with identical powers of ε, we have

$$\frac{1}{E}\sigma_{\alpha\alpha}=\varepsilon^{\varkappa+2}\sum_{s=0}\varepsilon^s\sigma_{\alpha\alpha}^{(s)}\quad(\alpha\beta),\qquad\frac{1}{E}\sigma_{\alpha\beta}=\varepsilon^{\varkappa+2}\sum_{s=0}\varepsilon^s\sigma_{\alpha\beta}^{(s)}\qquad(2.2)$$

$$\frac{1}{E}\sigma_{\alpha\gamma}=\varepsilon^{\varkappa+1}\sum_{s=0}\varepsilon^s\sigma_{\alpha\gamma}^{(s)}\quad(\alpha\beta),\qquad\frac{1}{E}\sigma_{\gamma\gamma}=\varepsilon^{\varkappa+2}\sum_{s=0}\varepsilon^s\sigma_{\gamma\gamma}^{(s)}\qquad(2.3)$$

where

$$\sigma_{\alpha\alpha}^{(s)}=\frac{\nu}{(1+\nu)(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\alpha^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha v_\beta^{(s)})\right]+$$

$$+\frac{1}{1+\nu}\left[\frac{1}{H_\alpha}\frac{\partial v_\alpha^{(s)}}{\partial\xi}-\frac{1}{H_\alpha H_\beta}\frac{\partial H_\alpha}{\partial\eta}v_\beta^{(s)}\right]+\frac{\nu}{(1+\nu)(1-2\nu)}\frac{\partial v_\gamma^{(s+2)}}{\partial\zeta}\qquad(\alpha\beta)\qquad(2.4)$$

$$\sigma_{\alpha\beta}{}^{(s)}=\frac{1}{2(1+\nu)}\left[\frac{H_\beta}{H_\alpha}\frac{\partial}{\partial\xi}\left(\frac{v_\beta{}^{(s)}}{H_\beta}\right)+\frac{H_\alpha}{H_\beta}\frac{\partial}{\partial\eta}\left(\frac{v_\alpha{}^{(s)}}{H_\alpha}\right)\right]$$

$$\sigma_{\alpha\gamma}{}^{(s)}=\frac{1}{2(1+\nu)}\left(\frac{1}{H_\alpha}\frac{\partial v_\gamma{}^{(s)}}{\partial\xi}+\frac{\partial v_\alpha{}^{(s)}}{\partial\zeta}\right)\qquad(\alpha\beta)$$

$$\sigma_{\gamma\gamma}{}^{(s)}=\frac{\nu}{(1+\nu)(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_\alpha{}^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha v_\beta{}^{(s)})\right]+$$
$$+\frac{1-\nu}{(1+\nu)(1-2\nu)}\frac{\partial v_\gamma{}^{(s+2)}}{\partial\zeta}$$

Let us substitute (1.5) in the obtained relationships, and let us combine terms in identical powers of ζ. We hence obtain

$$\sigma_{\alpha\alpha}{}^{(s)}=\sum_{k=0}^{r+1}\zeta^k\sigma_{\alpha\alpha k}^{(s)}\quad(\alpha\beta),\qquad \sigma_{\alpha\beta}{}^{(s)}=\sum_{k=0}^{r+1}\zeta^k\sigma_{\alpha\beta k}^{(s)}\tag{2.5}$$

$$\sigma_{\alpha\gamma}{}^{(s)}=\sum_{k=0}^{r}\zeta^k\sigma_{\alpha\gamma k}^{(s)}\quad(\alpha\beta),\qquad \sigma_{\gamma\gamma}{}^{(s)}=\sum_{k=0}^{r+1}\zeta^k\sigma_{\gamma\gamma k}^{(s)}\tag{2.6}$$

Here r is determined in conformity with (1.6). For the quantities with subscript k in (2.5) and (2.6) we have

$$\sigma_{\alpha\alpha k}^{(s)}=\frac{\nu}{(1+\nu)(1-2\nu)}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha k}^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha v_{\beta k}{}^{(s)})\right]+(k+1)\,v_{\gamma,\,k+1}^{(s+2)}\right\}+$$
$$+\frac{1}{1+\nu}\left[\frac{1}{H_\alpha}\frac{\partial v_{\alpha k}{}^{(s)}}{\partial\xi}-\frac{1}{H_\alpha H_\beta}\frac{\partial H_\alpha}{\partial\eta}v_{\beta k}{}^{(s)}\right]\qquad(\alpha\beta)$$

$$\sigma_{\alpha\beta k}^{(s)}=\frac{1}{2(1+\nu)}\left[\frac{H_\beta}{H_\alpha}\frac{\partial}{\partial\xi}\frac{v_{\beta k}{}^{(s)}}{H_\beta}+\frac{H_\alpha}{H_\beta}\frac{\partial}{\partial\eta}\frac{v_{\alpha k}{}^{(s)}}{H_\alpha}\right]\tag{2.7}$$

$$\sigma_{\alpha\gamma k}^{(s)}=\frac{1}{2(1+\nu)}\left[\frac{1}{H_\alpha}\frac{\partial v_{\gamma k}{}^{(s)}}{\partial\xi}+(k+1)v_{\alpha,\,k+1}^{(s)}\right]\qquad(\alpha\beta)$$

$$\sigma_{\gamma\gamma k}^{(s)}=\frac{\nu}{(1+\nu)(1-2\nu)}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha k}{}^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha v_{\beta k}{}^{(s)})\right]+$$
$$+\frac{1-\nu}{(1+\nu)(1-2\nu)}(k+1)\,v_{\gamma,\,k+1}^{(s+2)}$$

There are the following dependences

$$\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta\sigma_{\alpha\alpha k}^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha\sigma_{\alpha\beta k}^{(s)})-\frac{\partial H_\beta}{\partial\xi}\sigma_{\beta\beta k}^{(s)}+\frac{\partial H_\alpha}{\partial\eta}\sigma_{\alpha\beta k}^{(s)}\right]+$$
$$+(k+1)\,\sigma_{\alpha\gamma,\,k+1}^{(s+2)}=0\qquad(\alpha\beta)\tag{2.8}$$

$$\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta\sigma_{\alpha\gamma k}^{(s)})+\frac{\partial}{\partial\eta}(H_\alpha\sigma_{\beta\gamma k}^{(s)})\right]+(k+1)\,\sigma_{\gamma\gamma,\,k+1}^{(s)}=0\tag{2.9}$$

between the quantities $\sigma_{\alpha\alpha k}^{(s)}$, $\sigma_{\beta\beta k}^{(s)}$, $\sigma_{\alpha\beta k}^{(s)}$, $\sigma_{\gamma\gamma k}^{(s)}$, $\sigma_{\alpha\gamma k}^{(s)}$ and $\sigma_{\beta\gamma k}^{(s)}$, which are functions of ξ and η.

These dependences are easily obtained if the equilibrium equations in the stresses are used

$$\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\alpha}(H_\beta\sigma_{\alpha\alpha})+\frac{\partial}{\partial\beta}(H_\alpha\sigma_{\alpha\beta})-\frac{\partial H_\beta}{\partial\alpha}\sigma_{\beta\beta}+\frac{\partial H_\alpha}{\partial\beta}\sigma_{\alpha\beta}\right]+\frac{\partial\sigma_{\alpha\gamma}}{\partial\gamma}=0\qquad(\alpha\beta)\tag{2.10}$$

$$\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\alpha}(H_\beta\sigma_{\alpha\gamma})+\frac{\partial}{\partial\beta}(H_\alpha\sigma_{\beta\gamma})\right]+\frac{\partial\sigma_{\gamma\gamma}}{\partial\gamma}=0$$

If we transform to the nondimensional variables ξ, η and ζ in these equations, and take account of (2.2), (2.5) and (2.6), we then obtain (2.8) and (2.9). Let us note that if all the quantities in (2.8) and (2.9) are replaced by their expressions (2.7), it is easy to see that these relationships are satisfied identically.

3. Let us consider a homogeneous plate subjected to an arbitrary surface loading. The boundary conditions on the upper ($\zeta = \zeta_+$) and lower ($\zeta = \zeta_\pm$) planes are

$$\sigma_{\alpha\gamma}=\tau_\alpha^{\pm}(\alpha,\beta)\quad(\alpha\beta),\qquad \sigma_{\gamma\gamma}=\tau_\gamma^{\pm}(\alpha,\beta)\quad \text{for}\ \zeta=\zeta_\pm \tag{3.1}$$

Conditions (3.1) should be satisfied for the first nonzero members of the expansion (2.3) which we shall denote by s_0, i.e., for $\zeta = \zeta_\pm$ we have the conditions

$$E\varepsilon^{\varkappa+1+s_0}\sigma_{\alpha\gamma}^{(s_0)}=\tau_\alpha^{\pm}\quad(\alpha\beta),\qquad E\varepsilon^{\varkappa+2+s_0}\sigma_{\gamma\gamma}^{(s_0)}=\tau_\gamma^{\pm} \tag{3.2}$$

$$\sigma_{\alpha\gamma}^{(s)}=0\quad(\alpha\beta),\qquad \sigma_{\gamma\gamma}^{(s)}=0\quad \text{for}\ s>s_0 \tag{3.3}$$

Let us show that $s_0 = 2$, i.e., the first two members in (2.3) vanish.

Let us assume first that $s_0 = 0$, and let us elucidate the possibility of satisfying conditions (3.2). From (1.3), (1.5), (2.2), (2.3), (2.5), (2.6) it follows that for $s_0 = 0$ in zero-th approximation

$$v_\alpha^{(0)}=v_{\alpha0}^{(0)}+\zeta v_{\alpha1}^{(0)}\quad(\alpha\beta),\qquad v_\gamma^{(0)}=v_{\gamma0}^{(0)} \tag{3.4}$$

$$\sigma_{\alpha\alpha}^{(0)}=\sigma_{\alpha\alpha0}^{(0)}+\zeta\sigma_{\alpha\alpha1}^{(0)}\quad(\alpha\beta),\qquad \sigma_{\alpha\beta}^{(0)}=\sigma_{\alpha\beta0}^{(0)}+\zeta\sigma_{\alpha\beta1}^{(0)}$$

$$\sigma_{\alpha\gamma}^{(0)}=\sigma_{\alpha\gamma0}^{(0)}\quad(\alpha\beta),\qquad \sigma_{\gamma\gamma}^{(0)}=\sigma_{\gamma\gamma0}^{(0)}+\zeta\sigma_{\gamma\gamma1}^{(0)} \tag{3.5}$$

Six arbitrary functions $v_{\alpha0}^{(0)}$, $v_{\beta0}^{(0)}$, $v_{\gamma0}^{(0)}$, $\sigma_{\alpha\gamma0}^{(0)}$, $\sigma_{\beta\gamma0}^{(0)}$, and $\sigma_{\gamma\gamma0}^{(0)}$ enter into (3.4) and (3.5). The quantities $\sigma_{\alpha\gamma0}^{(0)}$, $\sigma_{\beta\gamma0}^{(0)}$ and $\sigma_{\gamma\gamma1}^{(0)}$ are connected by means of relationship (2.9) with $k = 0$.

Hence, the boundary conditions (3.2) may be satisfied only if the conditions

$$\tau_\alpha^{+}=\tau_\alpha^{-}\qquad(\alpha\beta) \tag{3.6}$$

$$\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta\tau_\alpha^{+})+\frac{\partial}{\partial\eta}(H_\alpha\tau_\beta^{+})\right]=-\frac{1}{2\varepsilon}(\tau_\gamma^{+}-\tau_\gamma^{-})$$

are imposed on the surface loading.

But by assumption the loading acting on the plate is arbitrary. Eliminating from consideration the particular case of a surface loading subjected to the conditions (3.6), we arrive at the deduction that it is impossible to satisfy the conditions (3.2) for $s_0 = 0$. Moreover, the $v_{\alpha0}^{(0)}$, $v_{\beta0}^{(0)}$ and $v_{\gamma0}^{(0)}$ remain undetermined. On this basis we conclude that $s_0 \neq 0$, i.e., the first members in (2.3) should vanish.

Now, let us assume that $s_0 = 1$. Then the quantities of the zero-th approximation are:

$$v_\alpha^{(0)},\ v_\beta^{(0)},\ v_\gamma^{(0)},\ \sigma_{\alpha\alpha}^{(0)},\ \sigma_{\beta\beta}^{(0)},\ \sigma_{\alpha\beta}^{(0)},\ \sigma_{\alpha\gamma}^{(1)},\ \sigma_{\beta\gamma}^{(1)}\ \text{and}\ \sigma_{\gamma\gamma}^{(1)}$$

As before, (3.4) holds for the first six of these, and we have for the last three

$$\sigma_{\alpha\gamma}^{(1)}=\sigma_{\alpha\gamma0}^{(1)}\quad(\alpha\beta),\qquad \sigma_{\gamma\gamma}^{(1)}=\sigma_{\gamma\gamma0}^{(1)}+\zeta\sigma_{\gamma\gamma1}^{(1)} \tag{3.7}$$

Six arbitrary functions

$$v_{\alpha0}^{(0)},\ v_{\beta0}^{(0)},\ v_{\gamma0}^{(0)}\ \sigma_{\alpha\gamma0}^{(1)},\ \sigma_{\beta\gamma0}^{(1)},\ \text{and}\ \sigma_{\gamma\gamma0}^{(1)}$$

are contained in (3.4) and (3.7). The quantity $\sigma_{\gamma\gamma1}^{(1)}$ is connected to $\sigma_{\alpha\gamma0}^{(1)}$ and $\sigma_{\beta\gamma0}^{(1)}$ by relationship (2.9) with $k = 0$. Hence, the boundary conditions (3.2) may be satisfied again only if the surface loading is subject to conditions (3.6). For $s_0 = 1$ the $v_{\alpha0}^{(0)}$, $v_{\beta0}^{(0)}$ and $v_{\gamma0}^{(0)}$ also remain uncertain. On the basis of all this it is clear that $s_0 \neq 1$, i.e., the second members in (2.3) also vanish.

Now let us show that arbitrary conditions with $\zeta = \zeta_\pm$ may be satisfied by setting $s_0 = 2$. The quantities

$$v_\alpha^{(0)},\ v_\beta^{(0)},\ v_\gamma^{(0)},\ \sigma_{\alpha\alpha}^{(0)},\ \sigma_{\beta\beta}^{(0)},\ \sigma_{\alpha\beta}^{(0)},\ \sigma_{\alpha\gamma}^{(2)},\ \sigma_{\beta\gamma}^{(2)} \text{ and } \sigma_{\gamma\gamma}^{(2)}$$

refer to the zero-th approximation for $s_0 = 2$. As before, we have (3.4) for the first six quantities, and for the last three

$$\sigma_{\alpha\gamma}^{(2)} = \sigma_{\alpha\gamma 0}^{(2)} + \zeta\sigma_{\alpha\gamma 1}^{(2)} + \zeta^2\sigma_{\alpha\gamma 2}^{(2)} \qquad (\alpha\beta)$$

$$\sigma_{\gamma\gamma}^{(2)} = \sigma_{\gamma\gamma 0}^{(2)} + \zeta\sigma_{\gamma\gamma 1}^{(2)} + \zeta^2\sigma_{\gamma\gamma 2}^{(2)} + \zeta^3\sigma_{\gamma\gamma 3}^{(2)} \tag{3.8}$$

Six arbitrary functions $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$, $v_{\gamma 0}^{(0)}$, $\sigma_{\alpha\gamma 0}^{(2)}$, $\sigma_{\beta\gamma 0}^{(2)}$ and $\sigma_{\gamma\gamma 0}^{(2)}$ enter (3.4) and (3.8). Substituting (3.8) into (3.2), we obtain six equations in six arbitrary functions. We can eliminate $\sigma_{\alpha\gamma 0}^{(2)}$, $\sigma_{\beta\gamma 0}^{(2)}$ and $\sigma_{\gamma\gamma 0}^{(2)}$ from these equations and obtain three differential equations in $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$ and $v_{\gamma 0}^{(0)}$. We consider this in greater detail in the next Section, but we note here that if $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$ and $v_{\gamma 0}^{(0)}$ are taken as the solution of the mentioned differential equations then conditions (3.2) can be satisfied for $s_0 = 2$. Therefore, $s_0 = 2$; this means that the expansions (2.3) start with the terms

$$\varepsilon^{\varkappa+3}\ \sigma_{\alpha\gamma}^{(2)},\quad \varepsilon^{\varkappa+3}\ \sigma_{\beta\gamma}^{(2)},\quad \varepsilon^{\varkappa+4} \text{ and } \sigma_{\gamma\gamma}^{(2)}$$

respectively, and the previous terms of the expansions vanish, that is

$$\sigma_{\alpha\gamma}^{(0)} = \sigma_{\alpha\gamma}^{(1)} = 0 \quad (\alpha\beta), \qquad \sigma_{\gamma\gamma}^{(0)} = \sigma_{\gamma\gamma}^{(1)} = 0 \tag{3.9}$$

If (2.7) is taken into account and also the relationship (2.9) with $k = 0$, it then follows from (3.9) that for $s = 0,\ 1$

$$\frac{1}{H_\alpha}\frac{\partial v_{\gamma 0}^{(s)}}{\partial \xi} + v_{\alpha 1}^{(s)} = 0 \qquad (\alpha\beta) \tag{3.10}$$

$$v_{\gamma 1}^{(s+2)} = -\frac{\nu}{1-\nu}\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha 0}^{(s)}) + \frac{\partial}{\partial\eta}(H_\alpha v_{\beta 0}^{(s)})\right] \tag{3.11}$$

The relationships (3.10) are equivalent to compliance with the Kirchhoff-Love hypothesis in the zero-th and first approximations.

4. Let us now obtain the differential equations for $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$, $v_{\gamma 0}^{(0)}$. Let us insert (2.6) for $\sigma_{\alpha\gamma}^{(2)}$, $\sigma_{\beta\gamma}^{(2)}$, $\sigma_{\gamma\gamma}^{(2)}$ into conditions (3.2), and then let us eliminate $\sigma_{\alpha\gamma 0}^{(2)}$, $\sigma_{\beta\gamma 0}^{(2)}$, $\sigma_{\gamma\gamma 0}^{(2)}$. We hence obtain three relationships

$$-(\zeta_+^2 - \zeta_-^2)\sigma_{\gamma\gamma 2}^{(2)} - 2(\zeta_+^3 - \zeta_-^3)\sigma_{\gamma\gamma 3}^{(2)} =$$

$$= \frac{1}{E\varepsilon^{\varkappa+4}}\left\{(\tau_\gamma^+ - \tau_\gamma^-) + \varepsilon\frac{1}{H_\alpha H_\beta}\left[\left(\frac{\partial(H_\beta\tau_\alpha^+)}{\partial\xi} + \frac{\partial(H_\alpha\tau_\beta^+)}{\partial\eta}\right)\zeta_+ - \right.\right.$$

$$\left.\left. - \left(\frac{\partial(H_\beta\tau_\alpha^-)}{\partial\xi} + \frac{\partial(H_\alpha\tau_\beta^-)}{\partial\eta}\right)\zeta_-\right]\right\} \tag{4.1}$$

$$2\sigma_{\alpha\gamma 1}^{(2)} + (\zeta_+^2 - \zeta_-^2)\sigma_{\alpha\gamma 2}^{(2)} = \frac{1}{E\varepsilon^{\varkappa+3}}(\tau_\alpha^+ - \tau_\alpha^-) \qquad (\alpha\beta) \tag{4.2}$$

It follows from (1.7), (1.8), (2.7), (3.10), (3.11) that

$$\sigma_{\gamma\gamma 2}^{(2)} = \frac{1}{2(1-\nu^2)}\nabla\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha 0}^{(0)}) + \frac{\partial}{\partial\eta}(H_\alpha v_{\beta 0}^{(0)})\right]\right\}$$

$$\sigma_{\gamma\gamma 3}^{(2)} = -\frac{1}{6(1-\nu^2)}\nabla\nabla v_{\gamma 0}^{(0)} \tag{4.3}$$

$$\sigma_{\alpha\gamma 1}^{(2)} = -L(v_{\alpha 0}^{(0)}, v_{\beta 0}^{(0)}) \quad (\alpha\beta), \qquad \sigma_{\alpha\gamma 2}^{(2)} = \frac{1}{2(1-\nu^2)}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\nabla v_{\gamma 0}^{(0)}$$

where the operator $L(v_{\alpha 0}^{(0)}, v_{\beta 0}^{(0)})$ is

$$L(v_{\alpha 0}^{(0)}, v_{\beta 0}^{(0)}) = \frac{1}{1-\nu^2}\frac{1}{H_\alpha}\frac{\partial}{\partial\xi}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\alpha 0}^{(0)}) + \frac{\partial}{\partial\eta}(H_\alpha v_{\beta 0}^{(0)})\right]\right\} -$$

$$- \frac{1}{2(1+\nu)}\frac{1}{H_\beta}\frac{\partial}{\partial\eta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial}{\partial\xi}(H_\beta v_{\beta 0}^{(0)}) - \frac{\partial}{\partial\eta}(H_\alpha v_{\alpha 0}^{(0)})\right]\right\} \quad (\alpha\beta) \qquad (4.4)$$

Substituting (4.3) into (4.1) and (4.2), we obtain three differential equations in $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$, $v_{\gamma 0}^{(0)}$. When the coordinate plane is disposed arbitrarily, all the unknowns enter into each of Eqs. (4.1) and (4.2). If the coordinate plane $\alpha\beta$ coincides with the middle plane of the plate then the bending and the generalized plane stress problems are separated. This is connected with the fact that the quantity $\zeta_+^2 - \xi_-^2$ vanishes in this case, only one unknown $v_{\gamma 0}^{(0)}$ will remain in (4.1), and only the unknowns $v_{\alpha 0}^{(0)}$ and $v_{\beta 0}^{(0)}$ in (4.2).

Henceforth, in considering a homogeneous plate we shall always assume that the coordinate plane $\alpha\beta$ coincides with its middle plane. In a zero-th approximation the bending equations are

$$\nabla\nabla v_{\gamma 0}^{(0)} = \frac{3}{2}(1-\nu^2)\frac{1}{E\varepsilon^{\varkappa+4}}\left\{(\tau_\gamma^+ - \tau_\gamma^-) + \right.$$

$$\left. + \varepsilon\frac{1}{H_\alpha H_\beta}\left[\left(\frac{\partial(H_\beta\tau_\alpha^+)}{\partial\xi} + \frac{\partial(H_\alpha\tau_\beta^+)}{\partial\eta}\right) + \left(\frac{\partial(H_\beta\tau_\alpha^-)}{\partial\xi} + \frac{\partial(H_\alpha\tau_\beta^-)}{\partial\eta}\right)\right]\right\} \qquad (4.5)$$

and the equations of generalized plane stress are of the form:

$$L(v_{\alpha 0}^{(0)}, v_{\beta 0}^{(0)}) = -\frac{1}{E\varepsilon^{\varkappa+3}}\frac{1}{2}(\tau_\alpha^+ - \tau_\alpha^-) \quad (\alpha\beta) \qquad (4.6)$$

We have homogeneous boundary conditions with $\zeta = \pm 1$ for the first approximation. Hence, the bending and generalized plane stress equations are also homogeneous, i.e.

$$\nabla\nabla v_{\gamma 0}^{(1)} = 0 \qquad (4.7)$$

$$L(v_{\alpha 0}^{(1)}, v_{\beta 0}^{(1)}) = 0 \quad (\alpha\beta) \qquad (4.8)$$

To obtain the bending and generalized plane stress equations in an s-th approximation, it is necessary to use the boundary conditions for this approximation with $\zeta = \pm 1$, to eliminate $\sigma_{\alpha\gamma 0}^{(s+2)}$, $\sigma_{\beta\gamma 0}^{(s+2)}$, $\sigma_{\gamma\gamma 0}^{(s+2)}$, and to express the remaining quantities in these conditions in terms of $v_{\alpha 0}^{(s)}$, $v_{\beta 0}^{(s)}$, $v_{\gamma 0}^{(s)}$ and in terms of values of the preceding approximations (which we consider known).

In an arbitrary orthogonal curvilinear coordinate system the bending problem in each approximation reduces to the solution of a biharmonic equation of the form:

$$\nabla\nabla v_{\gamma 0}^{(s)} = p_\gamma^{(s)} = \begin{cases} \nabla^{s/2}(a_s q_1 + b_s q_2) & \text{for even } s \\ 0 & \text{for odd } s \end{cases} \qquad (4.9)$$

where

$$q_1 = \frac{\tau_\gamma^+ - \tau_\gamma^-}{E\varepsilon^{\varkappa+4}} + q_2$$

$$q_2 = \frac{1}{E\varepsilon^{\varkappa+3}}\frac{1}{H_\alpha H_\beta}\left\{\frac{\partial[H_\beta(\tau_\alpha^+ + \tau_\alpha^-)]}{\partial\xi} + \frac{\partial[H_\alpha(\tau_\beta^+ + \tau_\beta^-)]}{\partial\eta}\right\} \qquad (4.10)$$

Here $\nabla^{s/2}$ denotes the polyharmonic operator of order $s/2$. Values of a_s, b_s have been obtained for $s = 0$, 2, 4 and 6. They are

$$a_0 = {}^3/_2\,(1+\nu)\,(1-\nu), \quad b_0 = 0$$

$$a_2 = -{}^3/_{20}\,(1+\nu)\,(8-3\nu), \; b_2 = {}^1/_2\,(1+\nu)\,(2-\nu)$$

$$a_4 = -{}^1/_{2800}\,(1+\nu)\,(227-157\,\nu), \quad b_4 = {}^1/_{20}\,(1+\nu)\,(1-\nu) \tag{4.11}$$
$$a_6 = -{}^1/_{252000}\,(1+\nu)\,(26-791\,\nu), \quad b_6 = -{}^1/_{8400}\,(1+\nu)\,(16+19\nu)$$

Values of a_s, b_s for $s = 0, 2$ had been obtained earlier [2].

The problem of the generalized plane stress reduces in each approximation to integration of the system of equations

$$L(v_{\alpha 0}^{(s)},\ v_{\beta 0}^{(s)}) = p_\alpha^{(s)} \qquad (\alpha\beta) \tag{4.12}$$

The right sides of these equations have been obtained for the first four approximations. They are of the form:

$$p_\alpha^{(0)} = -\frac{1}{E\varepsilon^{\varkappa+3}}\,\frac{1}{2}\,(\tau_\alpha^+ - \tau_\alpha^-) \quad (\alpha\beta), \qquad p_\alpha^{(1)} = 0 \quad (\alpha\beta)$$

$$p_\alpha^{(2)} = -\frac{1}{E\varepsilon^{\varkappa+3}}\,\frac{1}{12}\left\{\frac{2+\nu}{1+\nu}\,\frac{1}{H_\alpha}\times\right.$$
$$\times\frac{\partial}{\partial\xi}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial\,[H_\beta\,(\tau_\alpha^+ - \tau_\alpha^-)]}{\partial\xi} + \frac{\partial\,[H_\alpha\,(\tau_\beta^+ - \tau_\beta^-)]}{\partial\eta}\right]\right\} + \tag{4.13}$$
$$\left. + \frac{1}{H_\beta}\,\frac{\partial}{\partial\eta}\left\{\frac{1}{H_\alpha H_\beta}\left[\frac{\partial\,[H_\alpha\,(\tau_\alpha^+ - \tau_\alpha^-)]}{\partial\eta} - \frac{\partial\,[H_\beta\,(\tau_\beta^+ - \tau_\beta^-)]}{\partial\xi}\right]\right\}\right\} \quad (\alpha\beta)$$

$$p_\alpha^{(3)} = 0 \quad (\alpha\beta)$$

5. The states of stress for which the expansions (2.2) and (2.3) hold correspond to the asymptotic solution (1.3) of the Lamé equations. Hence, these states of stress are possible in a thin elastic layer. It has been shown above that a state of stress arises in a plate subject to an arbitrary surface loading, for which the expansions (2.2) will hold while the first two terms in the expansions (2.3) will vanish, in other words, the expansions (2.2) and (2.3) will hold for $s_0 = 2$. But in addition to such a state of stress, states of stress are possible in a thin layer for which there will hold: (a) expansions (2.2) and (2.3) with $s_0 = 1$ and (b) expansions (2.2) and (2.3) with $s_0 = 0$. Let us analyze each of the mentioned states of stress.

For the state of stress A to which the expansions (2.2) and (2.3) with $s_0 = 2$ correspond, it is characteristic that

1) The Kirchhoff-Love hypothesis is satisfied in the zero-th and first approximations;

2) The stresses $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$, $\sigma_{\alpha\beta}$ are the largest; the stresses $\sigma_{\alpha\gamma}$, $\sigma_{\beta\gamma}$ are one order, and the normal stresses $\sigma_{\gamma\gamma}$ two orders less than the fundamental stresses;

3) In a zero approximation the bending problem reduces to solving the customary equation of classical plate bending theory and determination of $v_{\alpha 0}^{(0)}$, $v_{\beta 0}^{(0)}$ reduces to the solution of the customary equations of the generalized plane state of stress.

The state of stress B which corresponds to the expansions (2.2) and (2.3) with $s_0 = 1$, has the following properties:

1) The Kirchhoff-Love hypothesis is satisfied only in a first approximations;

2) The stresses $\sigma_{\alpha\gamma}$, $\sigma_{\beta\gamma}$ are of the same order as $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$, $\sigma_{\alpha\beta}$ and the stresses $\sigma_{\gamma\gamma}$ are an order less;

3) In a zero-th approximation the former stresses are constant over the layer thickness.

The state of stress C corresponding to the expansions (2.2) and (2.3) with $s_0 = 0$ is characterized by the fact that:

1) The Kirchhoff-Love hypothesis does not hold even in the zero-th approximation;

2) The stresses $\sigma_{\alpha\gamma}$, $\sigma_{\beta\gamma}$ are the largest; the remaining stresses $\sigma_{\alpha\alpha}$, $\sigma_{\beta\beta}$, $\sigma_{\alpha\beta}$, $\sigma_{\gamma\gamma}$ are of an order less;

3) In the zero-th and first approximations the former stresses are constant over the layer thickness. This means that the Reissner hypothesis [4] is satisfied in the zero-th and first approximations for the considered state of stress.

The last two states of stress differ radically from the state of stress originating in a homogeneous plate subjected to arbitrary loading. It turns out that these states of stress in weak layers of laminated plates for which the ratio between the elastic moduli of the weak and stiff layers is commensurate with the relative thickness of the plate or with its square. Let us illustrate this by the example of a sandwich plate.

Let us first consider a sandwich plate for which the ratio between the elastic modulus of the filler E_1 and the elastic modulus of the stiff layers E_2 is commensurate with the relative thickness of the plate. The solution of the Lamé equations in both the domain occupied by the filler and the domains occupied by the stiff layers will be sought in the form (1.3).

Analyzing the possibility of complying with the boundary conditions on the upper and lower planes of the laminated plate, and with the static conditions of the combination of layers, we arrive at the deduction that the state of stress A corresponding to expansions (2.2) and (2.3) with $s_0 = 2$ originates in the stiff layers, and the state of stress B corresponding to (2.2) and (2.3) with $s_0 = 1$ in the filler. This means that the same asymptotic behavior holds in the stiff layers as has been considered in Sections 3 and 4, and an essentially different state of stress is produced in the filler. Let us analyze the state of stress in the filler in greater detail. Among the zero-th approximation quantities are the following

$$v_\alpha^{(0)},\ v_\beta^{(0)},\ v_\gamma^{(0)},\ \sigma_{\alpha\alpha}^{(0)},\ \sigma_{\beta\beta}^{(0)},\ \sigma_{\alpha\beta}^{(0)},\ \sigma_{\alpha\gamma}^{(1)},\ \sigma_{\beta\gamma}^{(1)},\ \sigma_{\gamma\gamma}^{(1)}$$

for which (3.4) and (3.7) with the six arbitrary functions:

$$v_{\alpha 0}^{(0)},\ v_{\beta 0}^{(0)},\ v_{\gamma 0}^{(0)},\ \sigma_{\alpha\gamma 0}^{(1)},\ \sigma_{\beta\gamma 0}^{(1)},\ \sigma_{\gamma\gamma 0}^{(1)} \tag{5.1}$$

hold.

Six geometric and six static matching conditions should be satisfied on the contact planes between the filler and the stiff layers. From these conditions we obtain twelve matching conditions for each approximation.

First we consider the zero-th approximation. Eliminating the six arbitrary functions (5.1) from the twelve matching conditions, we obtain six relationships between the zero-th approximation quantities for the stiff layers. We designate these relationships the compatibility conditions of the stiff layers in the zero-th approximation. The compatibility conditions of the stiff layers for the successive approximations are similarly obtained.

Let us now consider a sandwich plate for which the ratio E_1/E_2 is commensurate with the square of the relative thickness. Elucidating the possibility of complying with the boundary conditions on the upper and lower planes of a laminated plate and with the static conditions of the compatibility of layers, we arrive at the deduction that the state of stress A corresponding to (2.2) and (2.3) with $s_0 = 2$ holds, as before, in the stiff layers, while the state of stress C corresponding to (2.2) and (2.3) with $s_0 = 0$ holds in the filler. Hence, the following quantities:

$$v_\alpha^{(0)},\ v_\beta^{(0)},\ v_\gamma^{(0)},\ \sigma_{\alpha\alpha}^{(0)},\ \sigma_{\beta\beta}^{(0)},\ \sigma_{\alpha\beta}^{(0)},\ \sigma_{\alpha\gamma}^{(0)},\ \sigma_{\beta\gamma}^{(0)},\ \sigma_{\gamma\gamma}^{(0)},$$

for which (3.4) and (3.5) containing the six arbitrary functions

$$v_{\alpha 0}^{(0)},\ v_{\beta 0}^{(0)},\ v_{\gamma 0}^{(0)},\ \sigma_{\alpha\gamma 0}^{(0)},\ \sigma_{\beta\gamma 0}^{(0)},\ \sigma_{\gamma\gamma 0}^{(0)}$$

hold, refer to the zero-th approximation for the filler.

Eliminating these arbitrary functions from the twelve compatibility conditions for the layers in the zero-th approximation, we obtain six matching conditions for the stiff layers in the zero-th approximation.

In the problem of deformation of a sandwich plate, the matching conditions of the stiff layers permit elimination of the filler from the considerations. After the states of stress and strain in the stiff layers have been constructed, the determination of the displacements and stresses in the filler is associated only with the performance of algebraic operations and differentiation (without solving the differential equations).

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